Human Locomotion over Liquid Surfaces

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Using research on vertical water entry of circular disks and the basilisk lizard's locomotion over water, we investigate the theoretical minimum requirements for human movement over liquid surfaces. The goal is to develop a mathematical model which can be solved for the required running velocity, body mass, foot size, or the density of the liquid. First, we discuss previous research in related fields that is relevant to our model. We then analyze the interaction between the foot and the water surface and model the forces that act on the foot during each step. The subsequent study of three human feet allows us to estimate the impulse generated by the feet's impact on a water surface at different velocities. In order to execute the calculations, we assign a value to the maximum depth of the foot below the water surface during the step and express the stepping period as a function of velocity. Henceforth, we calculate the velocity that would allow an average person (m = 72kg, L_{foot} = 0.26m) to run over water (ρ = 1000kgm^{-3}) to be 35ms^{-1}. Finally we solve for the velocities that would allow human locomotion over other liquids with different densities.

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I. INTRODUCTION

In this paper, we investigate the theoretical minimum requirements that would allow humans to run over liquid surfaces. Given the density of a liquid as well as the individual's weight and foot size, we seek to determine the velocity needed to run over a liquid surface without sinking. Does a change in the density of the liquid greatly affect this velocity? If so, how dense would a liquid have to be for a human to run over its surface at realistic running velocities?

To answer these questions, we look at previous research on basilisk lizards. Young basilisk lizards are known to have the ability to run over water. The research on these lizards explores the forces that are generated during the lizard's step and act on the reptile's foot.

We then develop a similar model that describes the impulse generated by the human step on a liquid surface. We also determine an approximate allometric relationship between human foot length and foot size that simplifies the calculations.

It should be noted, however, that several assumptions have to be made throughout the paper in order to analyze the complex topic at hand. These assumptions, many of which are also used in the research on basilisk lizards, are needed to derive a mathematical model for human beings.

This paper is divided into five sections. Section II gives a brief overview of the previous literature on related topics. In section III we develop a mathematical model of the human step on a liquid surface. Section IV consists of a discussion of our results and section V contains a conclusion and suggestions for further research.

II PREVIOUS RESEARCH

A. Water running basilisk lizards

Basilisk lizards (*Basiliscus plumifrons*) (Fig. 1) are reptiles of the Amazon Rainforest. They have long whip-like tails and bodies covered with overlapping scales. The lizard uses camouflage to hide from its predators and has a life expectancy of about seven years.

Young basilisk lizards, specifically those with masses below 200g, have shown the ability to run over water without sinking. This ability provides them with a unique survival advantage, because the reptiles can use this skill to escape from their predators. The fact that only young lizards have the ability to run on water has important consequences for the population dynamics and densities of this species.

When basilisk lizards run over water, they generate an upward force by slapping the water surface with the foot. The foot is then pushed below the water surface, producing a ventilated air cavity. Finally, the lizard protracts its foot before the air cavity collapses. The step can consequently be divided into three different stages; slap (1), stroke (2) and protraction (3). It has been shown that the surface tension does not play an important in the locomotion of the lizards over water.[vi]

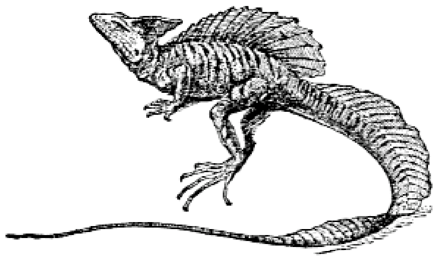

FIG. 1: A basilisk lizard (*Basiliscus plumifrons*). Young basilisk lizards have shown the ability to run over water without sinking.

When the foot of the lizard strikes the water surface (slap), a volume of water near the sole is suddenly accelerated downwards. As the leg and the foot continue to move downwards (stroke), fluid particles beneath the sole are displaced and an air cavity is formed above the foot. The protraction of the foot (3) within the air cavity minimizes friction, which would make the locomotion over water more difficult.

There are substantial size differences between juvenile and adult lizards. As mentioned earlier, aquatic locomotive capability among basilisks is size dependent; juveniles appear to run over water more easily than do adults. Grown lizards can often not produce an upward impulse large enough during the step to support their body weight for the period of the step.

Previous research has provided evidence that the difficulty of massive lizards to run over water is related to their inability to bring their feet back above the water surface before the air cavity collapses. This problem arises because larger lizards have to push their feet farther below the water surface to generate the necessary upward impulse.[1]

B. Vertical entry of disks on water

Glasheen and McMahon (1996a)[2] measured the forces that act on circular disks dropped into water at low Froude numbers (1 to 80)[3]. Similar to the step of basilisk lizard, the impact of the disks on the water surface can be divided into three distinct stages: (1) impact, (2) air cavity formation, and (3) cavity seal.

In their study, the authors used disks of various radii (r = 0.0127 to 0.0307 m) which were dropped into a tank of water. The disks were attached directly to an accelerometer that was fixed to a weight and a long rod. An optical encoder was used to measure the disk's change in velocity during an inelastic collision with no rebound. The authors found that the impact impulse is a function of both the initial and final velocities of the disk.[4]

Moreover, the authors found that the period of air cavity closures is linearly related to the square of the radius of the disk but is independent of the disk's velocity. We will utilize this result to model the human step on a liquid surface.

III. HUMAN LOCOMOTION OVER LIQUIDS

To examine the locomotion of humans over liquid surfaces, it is necessary to analyze the interaction between the foot and the liquid's surface during the step. In order to do this, we will enact certain simplifications. First, we will assume that the direction of the foot's velocity is perpendicular ($v_x = 0$) to the liquid's surface throughout the collision. Furthermore, we will suppose that the sole remains parallel to the undisturbed liquid surface during the entire step (the unrolling of the foot that takes place in locomotion over solid surfaces will not be considered). As a consequence of the first assumption, no frictional force in the x-direction will be generated.

Even though the motion of the foot is exclusively in the y-direction, the net force in the x-direction may not be zero. Some forces in this complex collision process may have components in the horizontal direction that do not cancel out because the human foot does not exhibit rotational symmetry. For our analysis, we will neglect any such possible forces in the horizontal direction, since we are only interested in forces that act orthogonally to the liquid's surface.

As mentioned earlier, the stepping process consists of three distinct stages (Fig. 2): The impact of the sole on the liquid's surface, which is followed by the air-cavity formation and the deceleration of the foot below the liquid's surface. Finally, the foot is protracted within the air cavity and the air cavity collapses. It is essential for human aquatic locomotion that the foot is protracted rapidly from the air cavity because drag forces would be too high to pull it up through the liquid.

Previous experiments that examined the force on circular disks dropped into a tank of water (Glasheen and McMahon, 1996a) found that the impulse generated during the impact stage is equal to the momentum of a sphere of water that has the same radius and velocity as the disk upon impact. We will use the same model to describe the impulse generated by a human foot striking a liquid surface:

$$I_{slap} = \int F_{slap} dt \qquad (1) \qquad (1)$$

$$I_{slap} = \frac{4}{3} r_{foot}^3 \rho v_1 \qquad (2) \qquad (2)$$

In this formula ρ is the density of the liquid and v_1 is the velocity of the foot upon impact. We define r_{foot} as the radius of a disk that would generate the same slap impulse as the foot if the disk traveled at velocity v_1. The radius r_{foot} can be described as a function of the foot length, but we will return to the development of this allometric relationship later.

To calculate the impulse that is generated during the stroke (stage 2), we will utilize Bernoulli's equation. The equation states that the total pressure along a streamline is the sum of the static pressure and the dynamic pressure. The contributions of the dynamic and hydrostatic pressure are given by $\frac{1}{2}\rho[v(t)]^2$ and $\rho g h(t)$, respectively. The force contribution of the hydrostatic pressure results from the creation of the air cavity and is equal to the weight of the displaced liquid.

To see why Bernoulli's equation is applicable in this context, we will consider an imaginary pipe running vertically downward below the foot's sole. During the stroke stage, the liquid is pushed vertically downward through this "pipe". By applying the equation, we neglect the fact that some liquid below the foot will also escape in the horizontal direction. The situation can, however, be approximated with Bernoulli's equation[5], because we are only considering a very short time interval (the time during which the air cavity exists). We can therefore find the upward force during the stroke :

$$P = \frac{1}{2}\rho[v(t)]^2 + \rho g h(t) \qquad (3)$$

$$F_{stroke} = -\left(\frac{1}{2}\rho[v(t)]^2 S + \rho g y(t) S\right) \qquad (4)$$

where P is the total pressure within the streamline, S is the area (πr_{foot}^2) of a disk of radius r_{foot} and $y(t)$ is the vertical distance of the foot below the liquid's surface. The vertical distance $y(t)$ is defined such that $\frac{dy}{dt} = v(t)$ and $y = 0$ at the liquid's surface.

Strictly speaking, Bernoulli's equation only applies if the flow along the streamline is inviscid (internal viscosity is zero), incompressible (constant pressure along the streamline) and steady. In particular, the steady flow assumption does not exactly hold for our model because the foot is decelerating during the stroke. Hence, we will introduce an additional drag coefficient (C) that estimates the effect of the foot's deceleration and other imperfections on the upward force (F_{stroke}) (Glasheen an McMahon, 1996b)[6]:

$$F_{stroke} = -C\left(\frac{1}{2}\rho[v(t)]^2 S + \rho g h(t) S)\right) \qquad (5)$$

The value of this dimensionless coefficient ultimately depends on empirical observation, and we will use an estimate based on the disk drop experiment by Glasheen and McMahon. The experiment determined the value of C to be approximately 0.702 regardless of the cavity depth and the disk's size and velocity.

The force F_{stroke} will act until the foot comes to a stop ($v_y = 0$) and the protraction begins. Since the foot is moving downward during the stroke, it will reach the lowest point below the liquid's surface at the start of the protraction stage.

Hence, we can estimate the total impulse if we assume constant deceleration during the stroke stage and substitute the average values for $v(t)$ and $h(t)$. The force F_{stroke} consequently acts over the time span of $\frac{L}{v_{avg}} = \frac{2L}{v_1}$, where L is the maximum depth of the foot below the liquid's surface. Using this information we can calculate the total impulse generated by the stroke force.

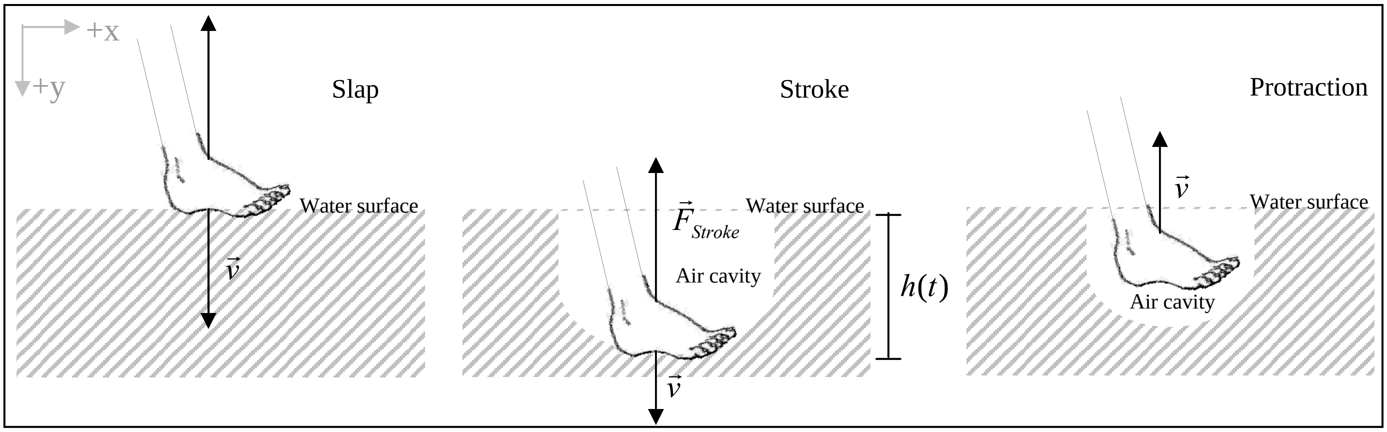

FIG. 2: the stepping process consists of three distinct stages: the impact of the sole on the liquid's surface, which is followed by the air-cavity formation and the deceleration of the foot below the liquid's surface. Finally, the foot is protracted within the air cavity and the air cavity collapses. It is essential for human aquatic locomotion that the foot is protracted fast from the air cavity because drag forces would be too high to pull it up through the liquid.

$$I_{stroke} = \underbrace{-C\left(\frac{1}{2}\rho\left(\frac{v_1}{2}\right)^2 S + \rho\left(\frac{L}{2}\right)gS\right)}_{F_{stroke}}\underbrace{\left(\frac{2L}{v_1}\right)}_{\Delta t}$$

$$I_{stroke} = -C\left(\frac{1}{8}\rho v_1^2 S + \left(\frac{1}{2}\right)\rho L g S\right)\left(\frac{2L}{v_1}\right) \tag{6}$$

In order to make continuous running possible, the impulses generated by the slap and the stroke need to be strong enough to keep the center of mass of the body at a constant height above the liquid. The minimum required impulse is consequently given by the body's weight times the stepping period (T_{period}):

$$I_{\min} = m_{body} g T_{period} \tag{7}$$

The condition for locomotion over liquid surfaces is therefore:

$$I_{\min} = I_{slap} + I_{stroke} \tag{8}$$

This equation can be solved for any of the input variables m, r_{foot}, v_1 or ρ.

IV. ANALYSIS OF RESULTS

In order to solve equation (8) we estimated the effective radius of the human foot. For this purpose we investigated three human feet (EU shoe size 40, 42, and 48). For each foot we measured the area of the sole and the length from the heel to the tip of the large toe. From this we calculated the radius (r_{foot}) of a circle that would have the same area as the sole. The calculations showed that the ratio of effective radius to foot length (in meters) is approximately constant $\frac{r_{foot}}{L_{foot}} \approx 0.3$ over the range of relevant foot sizes. Note that this estimation of the effective radius is not based on empirical observations of the impulse generated by human feet dropping in a tank of water.

Furthermore, it is necessary to assign a value to L, the maximum depth of the foot below the water surface. The maximum value of L is limited because the person must still be able to withdraw the leg before the air cavity collapses. In order to run, the person must also be able to align the second foot above the water surface for the next step while the first leg is partially submerged. Given these conditions, the length L must be considerably shorter than the human shin. For our calculations we use $L = 0.25m$.

In order to execute the calculations we express the stepping period as a function of the velocity: $T_{period} = \frac{2.3}{v_1}$. This estimate is based on Justin Gatlin's performance during the 2004 Summer Olympics. Gatlin completed the 100m race in 43 steps ($\approx 2.3 m/step$).

With these parameters, we calculate the required velocity v_1 that would allow a human ($m_{body} = 72kg$, $r_{foot} = 0.074m^{7}$) to run over water ($\rho = 1000 kg/m^3$) to be approximately

35msec^{-1}. The velocity required to run over the Dead Sea ($\rho = 1190\, kg/m^3$) for the same individual would be 32 msec^{-1}. The velocities that would permit human locomotion over other liquids (given the same input variables) are listed in Table 1.

TABLE I: Required velocities for human locomotion over different liquids

Liquid (density [kg m^{-3}])	Required velocity [m sec^{-1}] ([km h^{-1}])*
Crude Oil (915)	36 (130)
Bromine (3120)	20 (72)
Iodine (4927)	15 (54)
Mercury (13600)	10 (36)

Furthermore, it is possible to solve for the density of a liquid that in theory could permit locomotion at realistic running speeds. For a running speed $v_1 = 7ms^{-1}$ the liquid must have a density of at least $22850\, kg/m^3$, which is greater than that of the densest known liquid, mercury.

V. CONCLUSION AND FURTHER RESEARCH

The force analysis in the previous two sections clearly demonstrated why human locomotion is not suitable for movement over liquid surfaces. The area of the sole is too small to create an impulse great enough to support the mass of an average person at any reasonable velocity. Even for the densest known liquid (mercury), the foot would have to reach an impact velocity of 10ms^{-1}. This value is comparable to the velocity reached by world-class sprinters for a few seconds in the 100m race.

It is also important to note that throughout the analysis, we investigated the impact velocity of the foot and not the actual running (v_x^{COM}) velocity. Previous experiments showed that this velocity ($v_1 = v_1^y$) tends to be approximately 80-90 percent[8] that of the actual running velocity. This is partly caused by the fact that the foot in reality also has velocity in the x-direction when it meets the surface. The actual required running velocities for different liquids would therefore probably be even higher than the estimates presented in Table 1.

It is also important to note that we assumed that the direction of the foot's velocity is perpendicular to the liquid's surface throughout the collision process. Furthermore, we assume that the sole remains parallel to the undisturbed surface of the liquid throughout the step. These simplifications cause some distortion and do not accurately approximate human running. The assumptions are nevertheless appropriate for our purposes, because we are interested in investigating the minimum conditions that would allow human locomotion over water. Therefore, it is reasonable to expect the person to adjust his or her running style in order to maximize the impulse generated by the water.

Additionally, it is essential to emphasize again that the model developed in this paper is exclusively based on relevant previous research and not on any experiments that were specifically designed to investigate human locomotion over liquids. It would, for example, be valuable to experimentally determine the actual impulse generated by the vertical water (liquid) entry of human foot models. This would allow to find an empirical estimate of the effective radius (r_{foot}) of the human foot.

Further research could also investigate the effect of how the viscosities of liquids influence the upward force generated during the step. The potential stabilizing effect of viscosity is ignored in this paper because our mathematical model is based on Bernoulli's equation, which assumes zero internal viscosity. This is a good assumption for some liquids, such as water, but the validity of the assumption breaks down if the model is applied to very viscous media (e.g. oil). For viscous liquids it can be expected that the required running speed is lower than the estimates presented in Table 1.

Also note that as mentioned in Part two of Section one, it is not necessary to incorporate the effect of surface tension in the calculations. This assumption can be made because it has been shown that the impulse generated by surface tension is even negligible for very small animals, such as the basilisk lizards.

[1] Hsieh, S. Tonia, "Three-Dimensional Hindlimb Kinematics of Water Running in the Plumed Basilisk Lizard (Basiliscus Plumifrons)", Department of Organismic and Evolutionary Biology, Harvard University, 2003

[2] Glasheen, J. W. and T. A. McMahon, "Vertical Water Entry of Disks at Low Froude Numbers", American Journal of Physics, Vol. 8, No. 8, August 1996, pp. 2078-2083

[3] The Froude number gives the ratio of inertial forces to gravitational forces, or a dimensionless number that expresses the ratio of kinetic to potential energy.

[4] Impact impulse : $m_{missile} \Delta u$

[5] Bernoulli's equation can be derived from Euler's equation. For a derivation visit: http://en.wikipedia.org/wiki/Bernoulli%27s_equation

[6] Glasheen, J. W. and T. A. McMahon, "Size-Dependence of Water-Running Ability in Basilisk Lizard (Basiliscus Basiliscus)", Department of Organismic and Evolutionary Biology and Division of Applied Sciences, Harvard University, 1996

[7] This refers to a shoe size of 42. The Continental European shoe size can be converted in the length of the foot (in meters): $Shoe\ size = 150*(L_{foot}+0.02)$

[8] Rand, A. S. and H. Marx, "Running Speed of the Lizard Basiliscus Basiliscus on Water", Copeia, Vol. 1967, No. 1, Mar. 1967, pp. 230-233